Proxy Model Revisited: Critical Review of the “Fresh Look at the Proxy Model” by John E Farmer, Gina Gallant, and Norris Wilson

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Part I: Introduction

Calculations of historical loss of use in specific claims, where a First Nation has lost the use of land, can theoretically be approached using two scenarios:

> **Scenario I** assumes that, had the First Nation retained ownership of the claimed land, it would have operated it as a commercial venture. In this case, the net income from the venture would represent the First Nation's loss of use (LOU). The annual LOU values from the beginning of the loss to the present are then brought forward using multipliers.
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> **Scenario II** assumes that the First Nation would have leased the claimed land to a third party. The rental income the First Nation would have received, minus costs associated with leasing, would represent its LOU. These annual rental incomes are similarly brought forward using multipliers.

Calculation of the LOU based on the above two scenarios requires hypothetical historical data on what would have occurred had the First Nation retained ownership of the land. Such information is not readily available, and any attempt to estimate it would involve highly speculative assumptions, as well as significant time and cost.

Given the challenges in estimating historical losses using these scenarios, an alternative methodology—the Proxy Model—was developed by Professor Jack Carr in 2008. This approach aimed to be easy to implement, cost-effective, and transparent for all parties involved. However, the model suffered from issues related to land inflation and the calculation of rental rates of return, which I will address in this paper.

A revised approach was recently proposed to correct the shortcomings of the original model.[1] However, that report contains methodological errors that result in an overestimation of past LOU. The key issue with both approaches is their attempt to link the rental rate of return on land, a risky asset, to the return on financial instruments, which represent riskless assets. Additionally, both approaches assume a uniform rental rate of return for all land parcels in Canada, disregarding differences in productivity, climate, and other factors.

The main objective of this paper is to demonstrate that appraised land values—both historical and current—contain information about the rental rate of return, which can be extracted without relying on ad hoc assumptions or flawed proxies. This approach allows for a rental rate that is unique to each land parcel and reflects appraised land values, which are theoretically influenced by factors such as land productivity, climate, and other local conditions.

Additionally, this paper highlights issues with the use of proxies, the correlation coefficient concept, and assumptions made in the revised approach to the LOU calculation.

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[1] John E Farmer, Gina Gallant, and Norris Wilson, "Loss of Use Analysis: A Fresh Look at the Proxy Model", Canadian Property Valuation, Vol67|Book4/Tome4| 2023, pp. 26-31.

Part II: Issues with the Original Proxy Model

The Proxy Model developed by Professor Jack Carr in 2008 attempts to provide an alternative approach to calculating the LOU by suggesting the following approach:

1. Using the land value at the beginning of the loss period.
2. Constructing a time series of values for the land by multiplying the initial land value by the Consumer Price Index (CPI) during the period of loss.
3. Calculating the lost rent by multiplying the constructed land values by an assumed rental rate of 2% to 3%.
4. Calculating the present value of past rental losses using multipliers.

The attractive features of the Model are as follows:

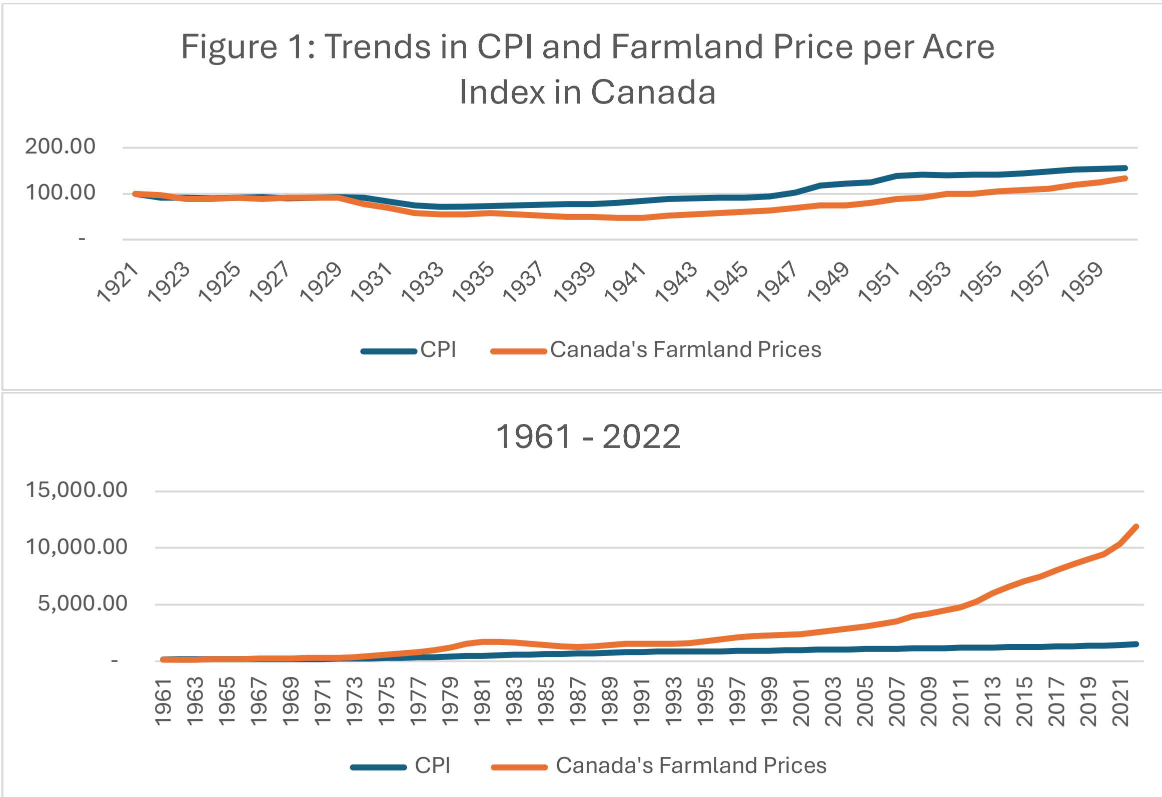
1. The Model is easy to implement.
2. It is transparent to all parties involved.
3. It is cost-effective as it saves time and reduces the cost associated with historical studies.
4. It assumes that rent is earned on every acre of land, and at every moment in time.
5. It assumes that there is no discount for lack of use of the land and there is no risk of not obtaining rent.

However, the original Proxy Model suffered from the following shortcomings:

1. Using the land value at the beginning of the loss period, the Proxy Model calculates historical land values from the beginning to the present time by updating the initial land value with the Consumer Price Index (CPI). The implicit assumption is that the nominal land prices follow the same trend as the CPI. Using Statistics Canada's farm price index from 1921 to the present and CPI, Figure 1 shows that this implicit assumption is not correct. The CPI slightly overestimates farmland prices from the 1930s to early 1960s and seriously underestimates them during the post-1960 period. Therefore, depending on the period of loss, the existing model can lead to either underestimation or overestimation of the LOU estimates.

2. The proposed rates of return (2.0% to 3.0%) used to calculate historical LOU in the existing Proxy Model are not based on the return to land but are taken from the real return on long-term government bonds during the 1930-2009 period. Real return on long-term government bonds declined significantly during 2008-2020, was negative in recent years, while rising again in the post-2022 period. Therefore, following the

existing model, the proposed rates of return can be challenged as the return on long-term government bonds changes.[2]



3. In practice, investors in land are not only interested in the rental rate of return but also tend to take into account any potential land price appreciation. In other words, investing in land is considered a hedge against inflation as well as providing some after-inflation yield in the form of rent or use value.

4. The accuracy of the rate of return is vital to the negotiating parties as it directly determines the annual LOU and thus the present value of the historical losses of use. Therefore, linking the rate of return to the return in the financial markets, which are influenced by various factors – including the Bank of Canada's monetary policy – is problematic.

5. In general, the return on long-term government bonds is affected by the Central Bank's monetary policy, and there is no one-to-one relationship between return to

[2] Studies show that long-term global real interest rates have exhibited a persistent downward trend of about 1.6 basis points per year. See: "The Real Interest rate Decline in Long Historical Perspective", National Bureau of Economic Research, No. 12, December 2022.

risk-free financial assets, such as bonds, and return to land, being a risky asset. Financial markets establish some sort of relationship between returns to different types of assets such as land and risk-free financial instruments. However, this relationship is not one-to-one.

Part III: Review of the Proposed "Loss of Use Analysis: A Fresh Look at the Proxy Model"

The authors of the "Fresh Look at the Proxy Model" correctly state that the principle on which a property's market value rests is that market value is the present value of the sum of expected future benefits arising from ownership of the land at any point in time. However, the authors err in the application of this concept, resulting in double-counting the rate of inflation and, therefore, over-estimating the loss of use. I first discuss the error in the author's model and the approach they have taken.

Issues in the "Fresh Look at the Proxy Model"

Given the land value at the beginning of the loss period as well as its current value, there is a need to estimate the intermediate prices between the two dates – historical and present. The authors recommend that the appraiser creates a historical timeseries of land prices using the farmland prices in Canada produced by Statistics Canada during the post-1921 period, and Kansas farm value data prior to 1921. They justify their choice of Kansas farm prices based on the argument that there is a strong correlation coefficient of 97% between the farm prices in Kansas and Canada during 1921-2019. I note that the authors mistakenly refer to R^2 as the correlation coefficient. The correlation coefficient ranges from -1 to +1. Clearly, R^2 cannot be negative as it is not the correlation coefficient. In their regression of Kansas prices on farmland prices in Alberta, R^2 shows the percentage of the variation in Kansas prices that is explained by the farmland prices in Alberta. R^2 does not say anything about the average prices in Kansas and Canada that are more relevant in the present context.[3]

Note that using the correlation coefficient concept in this context is incorrect. For example, consider the following two price series:

PA: 1,2,3,4,5,6,7,8,9,10
PB: 10,20,30,40,50,60,70,80,90,100

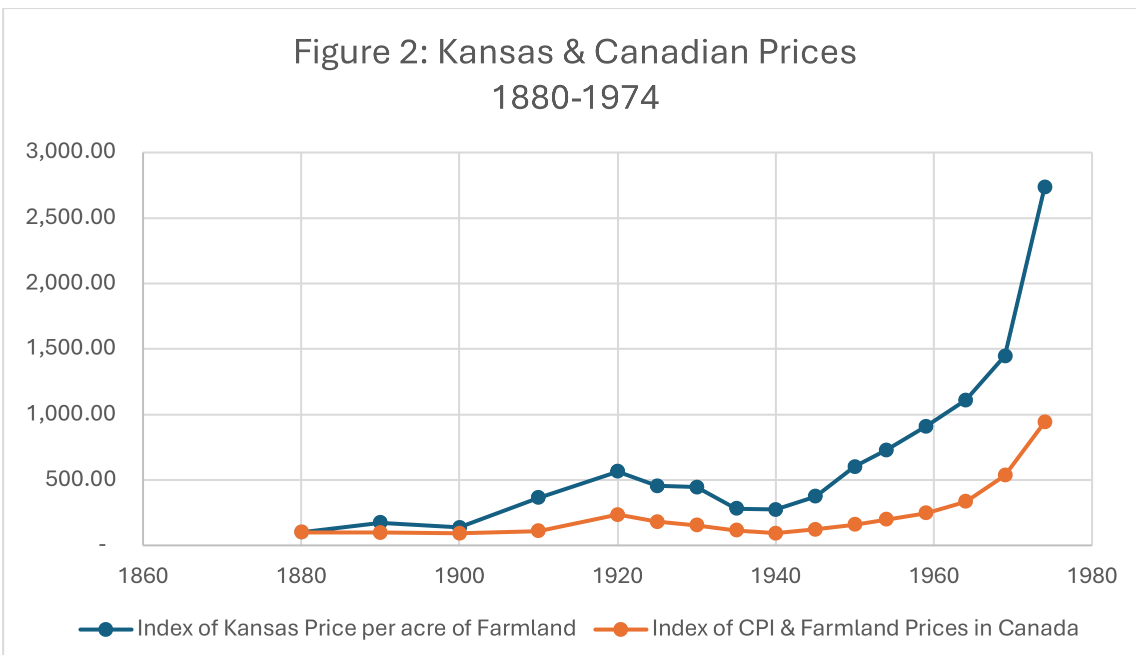
The price series PB is 10 times greater than PA. However, the correlation coefficient between the two series is 100%. Can one use PB as a proxy for PA? The answer is obviously negative. This is what the authors have done. I note that the combination of favorable climate, fertile soil, longer growing seasons, and large-scale mechanized farming operations makes Kansas

[3] I note that two series can be negatively related while showing a very high R^2.

one of the most productive agricultural regions in North America, and a powerhouse in crop and livestock production. Figure 2 compares Kansas price series with the Canadian ones. The Canadian price series is an index based on the CPI from 1880 to 1920 and farmland prices from 1925 to 1974. For comparison, I have set the initial values of both price indexes to 100.[4]

Figure 2 shows that the Kansas price series consistently exceeded the CPI-Farmland price series throughout the 1880-1974 period. Consequently, using Kansas farm prices to generate price series in various Canadian regions would lead to the overpricing of lands prior to the 1920s and thus overestimating the loss of use.

As Figure 1 shows, the CPI overestimates Canadian farmland prices prior to the 1960s. The CPI series is influenced by the Canadian economic conditions and thus would represent a better proxy than the Kansas series, which has no relation to the Canadian economy and its farmland prices. Moreover, the extent of the upward bias using the CPI is much less than using the Kansas series.

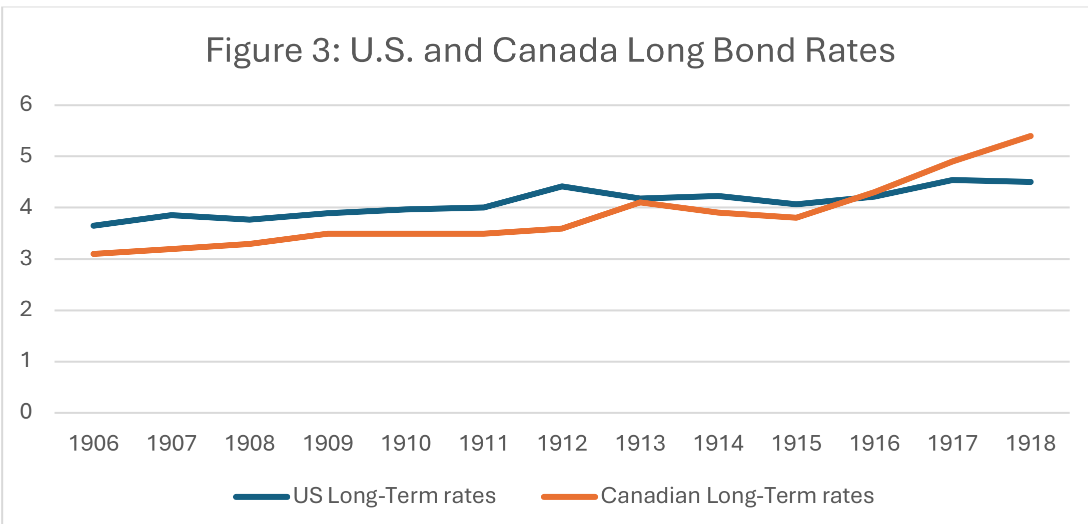


Using the same correlation argument between Kansas rental rates and nominal long-term bond interest rates in Canada in the post-1919 period, the authors suggest using nominal long-term bond interest rates as a proxy for rental rates in Canada in the post-1919 period. For the earlier period, the authors use the U.S. long-term bond interest rates as a proxy for

[4] Kansas price series is from Kansas State University, Manhattan Floyd W. Smith, director, "100 Years of Farmland Values in Kansas", September 1977.

the rental rates in Canada. Figure 3 shows that the U.S. long-term bond rates prior to 1919 were higher than the Canadian ones and thus would lead to overestimation of the loss of use.

I note that the nominal interest rate on a long-term bond includes two components.[5] First, is the expected rate of inflation (changes in CPI). As the expected inflation rate rises, nominal yield or return to bond holding rises as well. This is what has been happening to Government and corporate bond rates in the United States and Canada in recent years. The second component is the real return or after-inflation return as discussed in the original Proxy Model by Professor Jack Carr. Since the farmland prices already include inflation as a component, multiplying them by the nominal bond rates that also include the expected inflation results in double-counting of inflation. That is why Professor Carr suggests using after-inflation or real rate of return in the original Proxy Model to calculate lost rents.



In general, the rental rate represents a return to a risky asset. The bond rates represent a return to a risk-free financial asset when held to maturity. These are entirely different concepts and should not be used interchangeably based on some misconceived notion of correlation coefficient. There is no one-to-one relationship between the two. Additionally, the rental rate cannot be the same for all land parcels in Canada given different productivity levels in farmlands across the country. The following analysis shows that one can obtain a unique rental rate for each parcel of land based on the appraised initial and current value of the land in question. There is no need to use incorrect proxies and make ad hoc assumptions about the rental rates.

[5] The relationship is known as the Fisher equation and expressed as nominal interest rate = Real or after-inflation return + expected inflation.

Part IV: Underlying Theory of Market Prices and Rental Rates

What are the anticipated benefits to the owner of a real asset or, in general, any asset? Economic theory suggests that the total return or benefits to holding an asset (land, bonds, other types of assets) consists of two parts: First is the so-called "use value" in the form of rent, income (in kind or otherwise), or interest that one obtains from holding an asset. For example, one invests in government bonds to receive dividends at the end of each year. Alternatively, owning a real asset such as land provides an annual rent or "use value" in the form of agricultural products used for consumption or sale. The second component of total return is the capital appreciation (or depreciation) resulting from a change in the price of the asset. For example, assume one buys a one-year bond with a $1000 face value that pays $50 dividend after one year. If, during the holding period, the price of that bond rises to $1100[6], then the investor's total rate of return would equal:

Total return = (50/1000) + (1100 – 1000)/1000 = 5% + 10% =15%

The same is true for holding a real asset. Assume one buys a parcel of land for $1,000 and rents it for $50 per year. Also, assume inflation has been 10% during the year. The price of real assets rises with inflation. Therefore, their total return (TR) after a year is 15%. The real or after-inflation rate of return has been 5%.[7]

To formulate the above concept in a more precise way, I make the assumption that the land is owned at the beginning of time 't'. For simplicity, I also assume that the period of holding the land in question is one year. In other words, each person buys the land, holds it for one year, and sells it to another person.[8] As a real asset, the total nominal return to owning a land that is held from time 't' to time t+1 is composed of two elements. First is the inflation rate (Π) during that period, and the other is the rental return or after-inflation return or the use value return (R) during time 't'. Thus, the nominal return to investment for each investor who holds the land for only one year equals inflation plus rent. Therefore, given the purchase price of the land at time t, denoted as P_t, what would be the market price of that land at time t+1? The price of the land has risen during time t by inflation so equals $P_t \times (1+\Pi_t)$. The potential buyer expects to receive R percent of rent during period t+1. Thus, the potential buyer is willing to pay:

$$P_{t+1} = P_t \times (1+\Pi_t)(1+R_{t+1}) \qquad (1)$$

[6] This can happen if interest rates decline during the holding period resulting in an increase in the price of bonds.
[7] The exact relationship between the total return, inflation, and rental return is: $(1+TR) = (1+\Pi)(1+R)$.
[8] Note, there might be year-to-year changes in inflation and rental return, but the analysis holds over a long period of time, and therefore the inflation and rental rates can be considered as the average over an extended period of time.

The buyer expects the land to be a hedge against inflation during period t+1, and his net after-inflation return is expected to be R percent during period t+1. Equation (1) suggests that the market value of the land or the amount a potential buyer is willing to pay for the land at the beginning of time t+1 equals its market value at time 't' plus inflation during time 't' plus expected rental return during time t+1.[9]

Similarly, using the above numerical example, the market value of the land at the beginning of time t+2 equals:

$$P_{t+2} = P_{t+1} \times (1+\Pi_{t+1})(1+R_{t+2}) = \$115.5 \times (1.10)(1.05) = \$133.40 \quad (2)$$

Alternatively, assuming that, on average, the inflation and rental rates are Π and R, and substituting for P_{t+1} from (1) in (2) gives:

$$P_{t+2} = P_{t+1} \times (1+\Pi)(1+R) = P_t \times (1+\Pi)^2(1+R)^2 = \$100 \times (1.10)^2(1.05)^2 = \$133.40 \quad (3)$$

Equation (3) is the same as (2) with P_{t+1} being replaced by P_t.

Following the same procedure, we will find the market value of the land at time t+n to equal:

$$P_{t+n} = P_t \times (1+\Pi)^n(1+R)^n \quad (4)$$

Equation (4) tells us that if we have the market value of the land at the beginning of the loss period adjusted for land inflation during 't' to t+n period, ($P_t \times (1+\Pi)^n$), then we can calculate the average rental rate of return to land as:

$(1+R)^n = \{P_{t+n} / [P_t \times (1+\Pi)^n]\}$ or,

$R = \{P_{t+n} / [P_t \times (1+\Pi)^n]\}^{1/n} - 1$

R is the average rental rate of return or after inflation adjusted rate of return to land and is used as a proxy for the use value return. The above formulation suggests that, using the appraised market values, one can calculate a rental rate specific to the land in question.

In other words, there is no need to use proxies and ad hoc assumptions to calculate a rental rate that applies to all lands, as the market values already include information about the rental rate of return on a given parcel of land. Given differences in geography, climate, and soil condition, one cannot expect the rental rate to be the same for all lands.

[9] Returning to the above example: $P_{t+1} = \$100 \times (1.10)(1.05) = \$115.5$

Part V: Conclusion

This report shows that the appraised land prices at the beginning and end of the loss period contain information about the use value or rental rate of return to the land. The rental rates are unique to each parcel and can easily be obtained from the historical land values. This makes calculations of the loss of use straightforward without a need to use proxies and ad hoc assumptions.